NASA TECHNICAL TRANSLATION NASA TT F-11,663

NASA TT F-11,663

PROBLEMS ASSOCIATED WITH THE CALCULATION OF FRICTION AND HEAT TRANSFER IN A TURBULENT BOUNDARY LAYER

V. S. Avduyevskiy and V. N. Kalashnik

Translation of "Problemy Rascheta Treniya i Teploobmena v Turbulentnom Pogranichnom Sloye"
Izvestiya Akademii Nauk SSSR, Energetika i Transport, No. 5, pp. 9-24, 1967

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, DC 20546 APRIL 1968

SYMBOLS

u	velocity
α	coefficient of convective heat exchange
c_p	heat capacity at constant pressure
m_v	molecular weight of gas blown
δ_d	thickness of dynamic boundary layer
δ_t	thickness of thermal boundary layer
$\delta^* = \int_0^\delta [1-(\rho u/\rho_1 u_1)]\,dy$	displacement thickness of the boundary layer
$\theta_d = \int_0^\delta \left(\frac{\rho u}{\rho_1 u_1}\right)\left[1-\left(\frac{u}{u_1}\right)\right] dy$	momentum loss thickness of boundary layer
$\theta_1 = \int_0 \frac{\rho u}{\rho_1 u_1}\frac{T_{01}-T_0}{T_{01}-T_w}\,dy$	energy loss thickness (thermal energy) of boundary layer
$H = \frac{\delta^*}{\theta_d}$	form parameter
$H_{n=7}$	value of form parameter h for exponential profile with exponent $1/n = 1/7$
$H(x/l)$, $F(x/l)$	functions considering influence of geometric dimensions on coefficient of convective heat exchange with stepped change in surface temperature
T_e	restoration temperature
$r = \frac{T_e - T_1}{T_{01} - T_1}$	restoration coefficient
$Re_w = \frac{\rho_w u_1 x}{\mu_w}$	Reynolds number with respect to length from beginning of boundary layer and parameters on wall
$Re_{weff} = \frac{\rho_w u_1 x_{eff}}{\mu_w}$	Reynolds number with respect to effective length x_{eff} and parameters on wall
$Re_{\theta_d} = \frac{\rho_1 \mu_1 \theta_d}{\mu_1}$	Reynolds number with respect to momentum loss thickness and parameters on edge of boundary layer

Indices

0	deceleration parameters
1	parameters at edge of boundary layer
w	parameters on wall

NASA TT F-11,663

PROBLEMS ASSOCIATED WITH THE CALCULATION OF FRICTION AND HEAT TRANSFER IN A TURBULENT BOUNDARY LAYER

V. S. Avduyevskiy and V. N. Kalashnik

ABSTRACT. Discussion of the effect of the Mach number, pressure gradient, surface roughness, and similar parameters on the friction and heat transfer in a turbulent boundary layer. It is shown that the existing empirical formulas and semi-empirical methods for calculating these parameters, while based on experimentally determined friction and heat-transfer laws, are in poor agreement with experimental data in the case of large Mach numbers, compressible boundary layers on a rough surface, and certain other values of the parameters. Furthermore, semi-empirical methods do not permit extrapolation beyond the critical values for which the empirical relations (on which these methods are based) were derived. It is seen that, to improve this situation, it is necessary to refine the physical ideas on which these theories are based and to develop calculation methods applicable over a wider range of parameter values by using more rigorous physical relations for turbulent-boundary-layer flows.

The calculation of a turbulent boundary layer is performed in order to determine the coefficients of friction, heat and mass exchange on the surface of the body around which the flow occurs, as well as certain integral characteristics of the boundary layer: the displacement thickness, the impulse loss thickness, etc. At the present time, many various empirical and semi-empirical methods have been developed for calculating the turbulent boundary layer [1-8]. Attempts are being made to create calculation methods using deeper physical relationships between local characteristics in the boundary layer (turbulent viscosity, scale of turbulence, etc.) [9-12]. /9[1]

However, the existing calculation methods do not produce reliable results in all cases. We analyze below the specific features of flow in a turbulent boundary layer, which cannot be predicted using the available methods. Some of the problems will be simply stated.

I. Turbulent Boundary Layer on a Flat Plate

A great number of experimental works have been dedicated to the study of the boundary layer on a plate, and these works have allowed the primary specific

[1] Numbers in the margin indicate pagination in the foreign text.

features of this type of flow to be determined. The achievements of the semi-empirical theories of turbulent boundary layers in an incompressible liquid are well-known: these achievements include the production of universal logarithmic velocity profiles. The semi-empirical methods of calculation were further developed in recent years by the development of the well-known calculation methods of Prandtl-Carmen, Loytsyanskiy, Kalikhman, Iyevlev, von Driest, Wilson, Ginzburg and others. As experimental data have been accumulated on the influence of the primary flow parameters (Mach number, temperature factor, pressure gradient, etc.) on the development of a turbulent boundary layer, these methods are made ever more precise. It should be noted that the hypotheses on which these semi-empirical theories are based may not correspond in every detail with the physical picture of flow in a boundary layer. Therefore, these theories should be looked upon as methods for producing a form of calculation dependence which must then be clarified using experimental data. /10

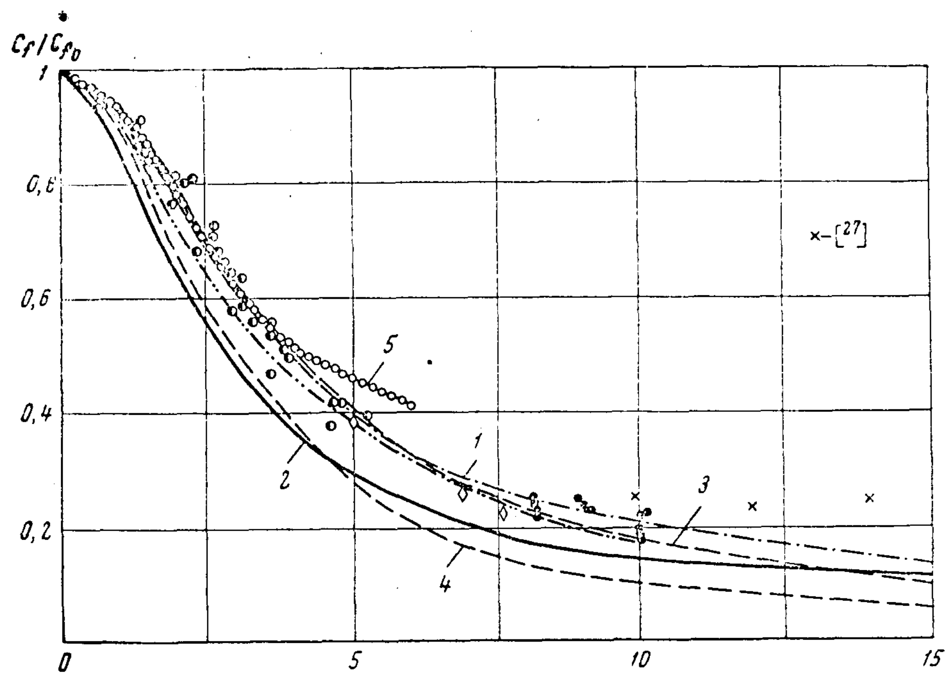

Figure 1. Influence of M Number on Coefficient of Friction for Heat Insulated Surface for Re_{θ_d} = const:

1, Calculated [3]; 2, Calculated [4]; 3, Calculated [5] with Re_{θ_d} = 2000; 4, Calculated [5] with $Re_{\theta_d} \to \infty$; 5, Calculated [8]. Experimental points from works of various authors.

As the methods for calculating an incompressible boundary layer were extended to cover the flow of a compressible gas, it was necessary to consider the change in physical properties (density ρ, viscosity μ, heat conductivity λ, etc.) across the boundary layer. In the works mentioned above, the methods of calculating the compressible layer were constructed on the basis of the ordinary conceptions of the semi-empirical theory, but in consideration of the variability of ρ, μ and λ across the layer. On the whole, the consideration of the variability of the physical properties makes it possible to indicate the influence of compressibility correctly. However, for a plate the same results could be produced by selecting mean values of these parameters (ρ, μ, λ, etc.) and using them directly in the final empirical formulas (for example, [4]).

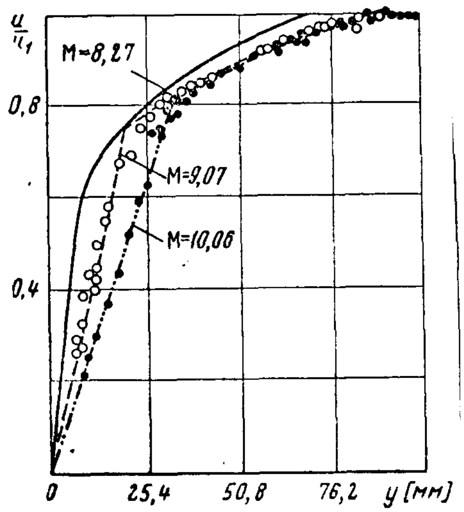

Figure 2. Influence of M Number on Form of Velocity Profile [17]

Figure 1 shows the experimental values of coefficient of friction C_1 with various Mach numbers on a heat insulated surface [13-16]. We can see that the calculation dependences of C_f/C_{f_0} generally quite properly reflect the influence of the M number on C_f where $M < 9$, since /11 in this area many measurements have been performed which can be used to clarify the theories. With greater M numbers, the experimental points lie above the calculated curves.

As yet, insufficient data have been accumulated to make a final judgment concerning the behavior of a turbulent boundary layer in the area of large Mach numbers; however, we should know the experimental facts indicating the changes in principle in the nature of the flow in a boundary layer at large Mach numbers. We can see from Figure 2 that the velocity profiles with increasing M number

$$\overline{\mathrm{Nu}}_{w\,\mathrm{eff}} = \frac{\mathrm{Nu}_{w\,\mathrm{eff}}}{(T_w/T_e)^{0,39}(\theta_T/\theta_{\text{Д}})^{1/10}\,\mathrm{Pr}_w^{0,48}\left(1 + r\frac{k-1}{2}\mathrm{M}_1^2\right)^{0,11}}$$

are deformed [17], the thickness of the laminar sublayer increases, reaching 20-30% of the thickness of the boundary layer, and its influence on all characteristics of the boundary layer increases. It is possible that this is the reason why extension of the semi-empirical theories and limit rules to the area of large Mach numbers does not give satisfactory results.

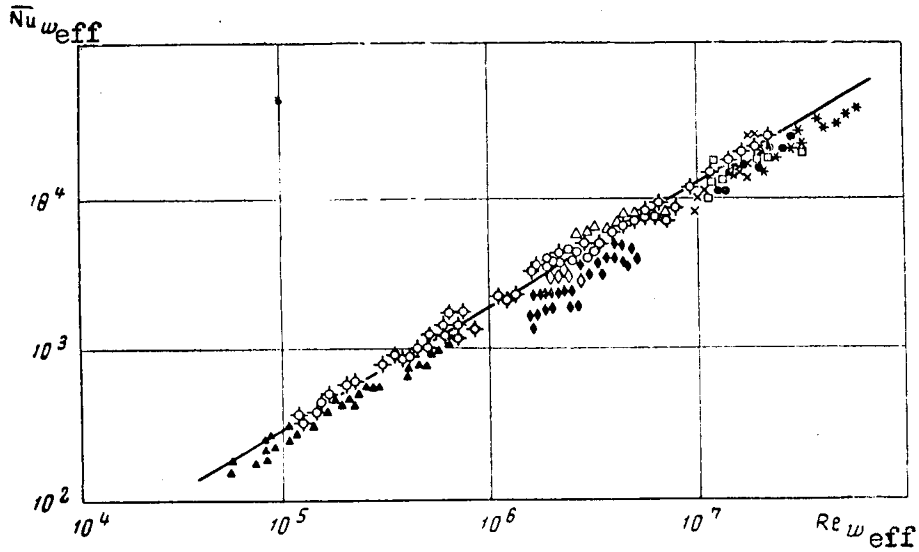

Figure 3. Comparison of Experimental Data on Heat Exchange in Nozzles at M = 5.5-10, on a Plate at M = 0-4 and on a Cone, Calculated According to Formulas (1) and (2)

II. Boundary Layer with Pressure Gradient

1. Negative pressure gradient. The results produced for a plate can be extended to a flow with weak longitudinal pressure gradient. The existing calculation methods are based on the assumption that the local characteristics of the boundary layer C_f, $St = \alpha/(\rho_1 u_1 c_p)$, $H = \delta^*/\theta_d$ depend on local values of the Reynolds number through the momentum loss thickness $Re_{\theta_d} = \rho_1 u_1 \theta_d / \mu_1$ just as on a plate; this means that the influence of the local pressure gradient is not taken into consideration. Over a certain range of pressure gradients ($0 > G > -10^{-3}$, where $G = \{\theta_d/(\rho_1 u_1^2)\}dp/dx$), this approach gives satisfactory results. We can cite as an example the method of the effective length for the calculation of heat exchange [8]. The essence of this method is that for each cross section on the surface of the body being examined, a plate of length x_{eff} is selected, over which an identical boundary layer develops. Figure 3 shows the experimental values of the coefficient of heat exchange produced under various conditions. The effective length is determined from the formula /12

$$x_{\text{eff}} = \frac{\int\limits_0^x (p_1/p_{01})^{\beta} M_1 R^{5/4} (T_e - T_w)^{9/8} (T_w/T_e)^{0,49} (\theta_d/\theta_T)^{1/8} dx}{(p_1/p_{01})^{\beta} M_1 R^{5/4} (T_e - T_w)^{9/8} (T_w/T_e)^{0,49} (\theta_d/\theta_T)^{1/8}} \tag{1}$$

$$\left(\beta = \frac{1{,}363 - 0{,}363k}{k}\right),$$

and the coefficient of heat exchange α is determined using the empirical formula for a flat plate [4]

$$\mathrm{Nu}_{w\,\text{eff}} = \frac{\alpha x_{\text{эфф}}}{\lambda_w} = 0{,}0296\,\mathrm{Re}_{w\,\text{eff}}^{0,8} \left(\frac{T_w}{T_e}\right)^{0,39} \mathrm{Pr}_w^{0,46} \left(1 + \frac{k-1}{2} r\mathrm{M}^2\right)^{0,11} \tag{2}$$

It is interesting to note that the usage in these formulas (as well as in formulas of other methods of calculation of heat exchange) of enthalpy I in place of the corresponding temperatures allows us to produce rather reliable results in calculating the turbulent boundary layer in a dissociated gas.

The effective length method for calculating heat exchange allowed us to use the experiments performed in flows with pressure gradients (primarily in nozzles) to clarify the empirical dependences in the area of large M numbers, low values of temperature factor, etc.

The analogous method for the calculation of friction and dynamic boundary layer requires more detailed information on the form of the velocity and temperature profiles. The ordinary assumption that the set of velocity profiles has one parameter and that this parameter is $H = f(M_1, Re_{\theta_d}, T_w/T_c)$ gives fair results when the following empirical dependences are used

$$H = \frac{9}{7}\left\{\left(1 + \frac{k-1}{2}\mathrm{M}_1^2\right)\left[\frac{T_w}{T_{01}} + \left(1 - \frac{\delta_T}{\delta_d}\right)\left(1 - \frac{T_w}{T_{01}}\right) + \frac{k-1}{2}\mathrm{M}_1^2\right]\right\} \tag{3}$$

$$\frac{H}{H_{n=7}} = 1{,}12\left[\frac{\mathrm{Re}_{\theta_d}}{1000}\right]^{-0,04}. \tag{4}$$

However, the usage of the law of resistance without consideration of the influence of the local pressure gradient on friction leads to erroneous results in the case of large pressure gradients in the flow.

With a negative pressure gradient, the velocity profile is deformed so that friction and heat exchange increase with identical layer thicknesses. This effect is either taken into consideration in certain calculation methods based on experimental data, or is included in the semi-empirical calculation methods in determining the distribution of the shear stress across the layer [18, 19]. However, in accelerating flows there is yet another factor which exerts its influence in the opposite direction. We are speaking of the suppression of turbulence and corresponding decrease in transfer coefficients. As an example, we can cite works [20, 21] in which it is shown that in the presence of a strong negative gradient in nozzles, laminarization of the turbulent boundary layer occurs. This fact is not predicted in principle by the semi-empirical theories and requires more precise methods.

In work [22], measurements of the coefficient of heat exchange on a plate were performed for subsonic velocities in the presence of sectors with negative /13
pressure gradients (Figure 4). With sufficiently large pressure gradients, the experimental points were notably lower than the calculated dependence for a turbulent boundary layer. This work suggested parameter $K = -G/Re_{\theta_d}$ and included a determination of the value of K_{max}, after achievement of which a decrease in the heat exchange coefficient is noted. However, at the present time there are no methods for calculating the turbulent boundary layer under these conditions.

When a supersonic boundary layer develops over an angular point, the effects analyzed above also appear, i.e. deformation of the velocity profile occurs, the profile becoming fuller, and turbulent pulsations in pressure occur. However, it should be noted that the boundary layer equation is not followed in the immediate vicinity of the angular point, since there is a large transverse pressure gradient at that point.

Figure 5 shows experimental values of heat exchange on a cylinder beyond the angular point [23]. We see that the calculated heat exchange from the critical point disagrees with the experimental value only in the area of the angular point. Calculation of the heat exchange on the assumption that the boundary layer begins to develop at the angular point gives an artificially high value of α which, however, approaches the experimental value at large distances. Good correspondence with the experimental values is observed when calculations are performed in consideration of deformation of the velocity profile upon rotation [23]. Rotation of the boundary layer can be calculated using the formulas for an ideal gas without considering the influence of viscosity. After the rotation, development of a new boundary layer is analyzed in the flow with transverse velocity gradient. In this case, the existence of a laminar boundary layer in the area of the angular point is possible. At sufficiently large distances from the angular point, the influence is reduced, and the three calculation dependences converge. It should be noted that all the above is correct for those cases when the flow moves past the angular point practically without separation.

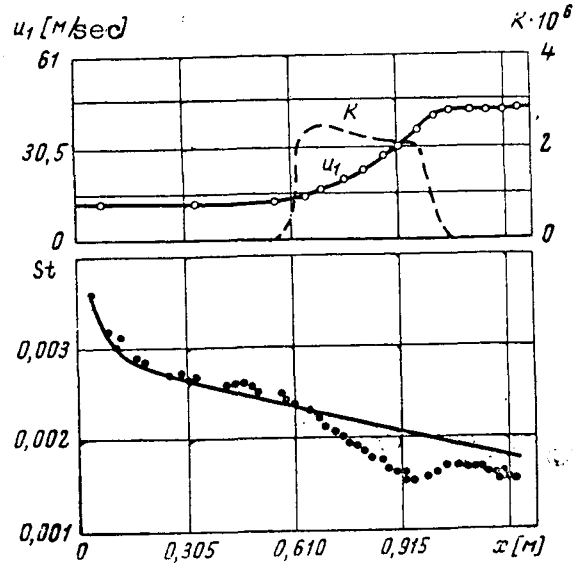

Figure 4. Influence of Negative Pressure Gradient on Heat Exchange Coefficient in Turbulent Boundary Layer. Before sector with negative pressure gradient $Re_{\theta_d} = 1890$ and

$Re = 0.77 \cdot 10^6$ [22]

2. Positive pressure gradient. Flows with positive pressure gradients have always been studied deeply. Many works have been dedicated to the development of integral methods of calculating the boundary layer for the case $dp/dx > 0$ [11, 19, 24, 25, etc.] In this case, very strong deformation of the velocity profiles occurs, particularly when we approach the point of separation. However, the assumption that the set of velocity profiles has one parameter gives satisfactory results. As a form parameter, sometimes the value of H_i is used [25, 26] /14

$$H_i = \int_0^\delta \left[1 - \frac{u}{u_1}\right] dy \Big/ \int_0^\delta \frac{u}{u_1}\left[1 - \frac{u}{u_1}\right] dy.$$

We can see from Figure 6 that with identical values of this parameter, the velocity profiles produced under various conditions agree well with each other.

Many dependences have been suggested for the resistance rule, of which the most successful is the Ludvieg-Tillmann formula [18].

In [25], a method is suggested for calculating friction in a turbulent boundary layer with pressure gradient; in this method, the Ludvieg-Tillmann formula is used as a resistance rule, and the form parameter selected is H_i. The following formula was produced for C_f:

$$\frac{1}{2} C_f = \frac{\tau_w}{\rho_1 u_1^2} = 0{,}123 e^{-1{,}561 H_i} \left(\frac{\rho_1 u_1 \theta_d}{\mu_1}\right)^{-0{,}268} \left(\frac{T_0}{\overline{T}}\right) \left(\frac{\overline{\mu}}{\mu_0}\right)^{0{,}268}. \tag{6}$$

On Figure 7, taken from [25], we see that calculation using this formula agrees qualitatively with the experimental values for a complex form body, the /15
surface of which has areas both with positive and with negative pressure gradients. However, the quantitative correspondence for C_f is unsatisfactory, although the calculated and experimental values of momentum loss thickness

correspond. This is related to the fact that the formula for this type of resistance rule is not universal, and the set of velocity profiles can be considered one-parameter only in the first approximation.

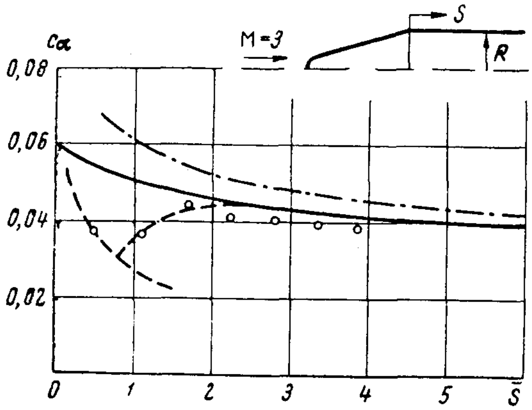

Figure 5. Influence of Rotation about Angular Point on Heat Exchange Coefficient in Turbulent Boundary Layer. Coordinate S measured from angular point [23]. $\bar{S} = S/R$

In supersonic flows, calculation of the boundary layer is complicated due to the reverse influence of the boundary layer on the external free flow. This problem goes beyond the framework of the boundary layer theory. However, from experiments on the interaction of the boundary layer with compression jumps, we can produce additional information for the resistance rule and the value of form parameter H at supersonic velocities on small models. In these experiments, it is easy to determine the displacement thickness δ^* with various values of dp/dx and the position of the point of separation with respect to the jump using shadow photography. The quantities determined allow the value of G to be determined at the separation point. This is the simplest and most precise method of determining separation values of G in supersonic flows.

III. Area of Transition to Turbulent Flow in Boundary Layer

Calculation of the position of the zone of transition of a boundary layer and the changes in the nature of the layer in this zone is an extremely important task which has not yet been solved. The problem is complicated by the large number of parameters which influence the transition of the boundary layer strongly.

Although a great number of experimental works have been dedicated to this transition, as yet no systematic data have been developed on the influence of each parameter individually.

It can be considered established at the present time that the Reynolds number for transition on a heat insulated surface with dp/dx = 0 increases monotonously with increasing Mach number for $M > 3$ [27, 28]. It has also been established that the length of the transition zone may be very great and at supersonic velocities the plate sector covered by the transitional layer may be greater than the sector covered by laminar flow [27, 29]. It is known that cooling of the surface may essentially delay onset of the transition on a surface at Mach numbers M = 2-3 [30].

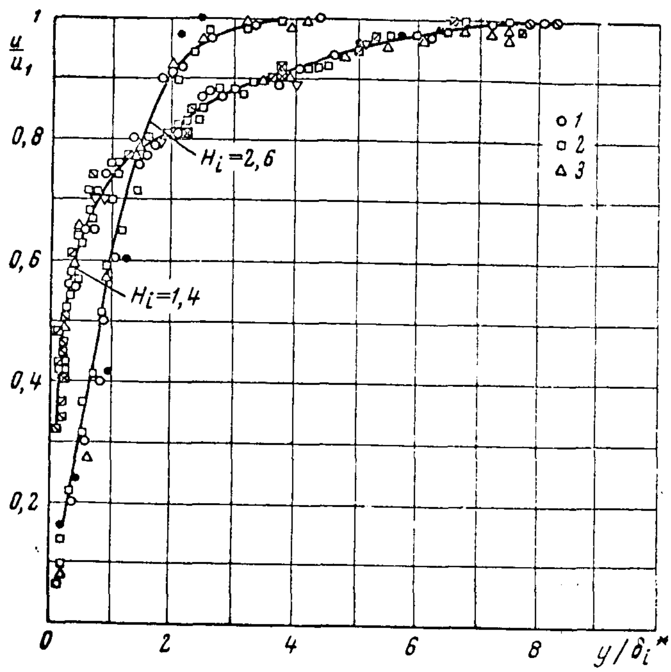

Figure 6. Comparison of Velocity Profiles Produced with Various M Numbers and Identical Values of Form Parameter H: 1, M = 1.42; 2, M = 1.85-2.96; 3, M = 5

Concerning heat exchange, it has been established that in the transition area the coefficient of heat exchange increases monotonously from laminar to turbulent values. For a flat plate, the following relationship between the Stanton number in the transition area and the Reynolds number has been established (Figure 8):

$$St_w = A\,Re_w{}^n \quad (n = 0{,}8\text{—}1{,}0), \tag{7}$$

where A depends on the Reynolds number at the beginning of transition.

The primary difficulty involved in calculating heat exchange in the transition area is related to a determination of the position of the transition zone on the body being analyzed. /16

Measurement of friction in the transition area has practically never been performed. Where necessary, friction in the transition area is estimated using the Reynolds analogy on the basis of the available experimental data on heat exchange.

It should be noted that roughness in the transition area may cause a considerable increase in the coefficient of heat exchange and particularly the coefficient of temperature restoration r. Whereas with natural transition of a boundary layer over a smooth surface the coefficient of restoration increases from the laminar value of 0.84-0.85 to the turbulent value r = 0.88-0.90, over a rough surface the values of r may reach 0.95-0.98.

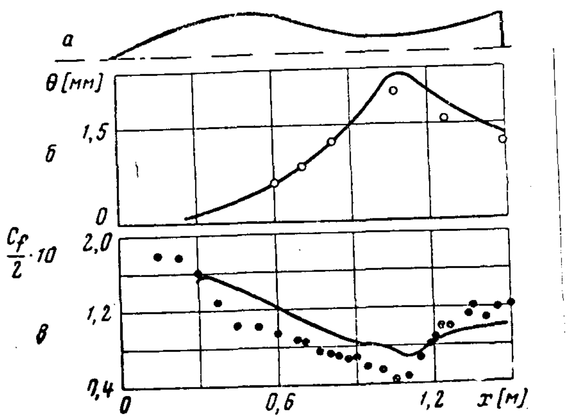

Figure 7. Calculation of Coefficient of Friction C_f and Momentum Loss Thickness θ_d on Complex Form Body with Mach Number of Incident Flow 2.0; a, Shape of body; b, Comparison of calculated and experimental values of θ_d; c, Comparison of calculated and experimental values of C_f

IV. Boundary Layer on a Rough Surface

Based on the classical experiments by Nikuradse [31] conducted in rough tubes, a method has been developed for calculating friction over rough plates around which an incompressible liquid flows [32]. The concept of the equivalent sand roughness was developed, which allowed the form of the roughness elements to be eliminated from the analysis. It was found that the ratio of the height of the equivalent sand roughness to the thickness of the laminar sublayer is a parameter which characterizes the degree of influence of the roughness on friction. When the height of the roughness is less than the thickness of the sublayer, the roughness of the surface does not influence friction, and the surface can be considered smooth. When the height of the roughness is considerably greater than the thickness of the laminar sublayer, the resistance of the form of the roughness becomes determinant; at this point the value of C_f ceases to depend on the Reynolds number, and depends only on the height of the roughness. In the intermediate area, C_f depends both on the height of the roughness and on the Reynolds number.

The method of calculating C_f in an incompressible turbulent boundary layer over a flat plate [32] agrees well with experimental data, as we can see from Figure 9, which shows experimental [33] and calculated values of the mean coefficient of friction C_F on a plate of length l at M = 0.7. /17

Many attempts have been undertaken to consider the influence of compressibility; however, no satisfactory method for calculating friction in a compressible layer over a rough surface has ever been produced. On Figure 10 we see experimental values of C_F for a rough surface of length l at M = 2, as well as calculated values of C_F which do not consider the influence of compressibility [32] and which do consider the influence of compressibility [34]. We see that the divergence between the experimental and calculated values remains considerable. It should be noted that with increasing M number, the influence of roughness on friction decreases.

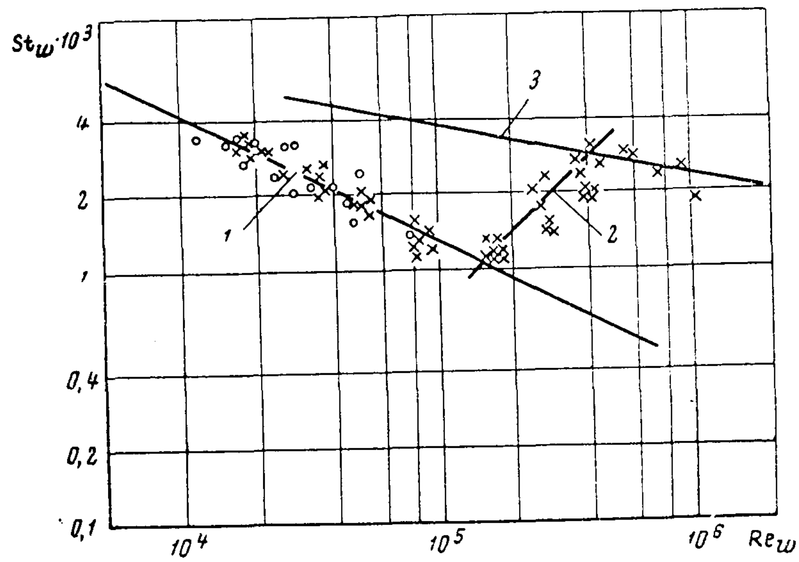

Figure 8. Experimental Values of Heat Exchange Coefficient in Gradient-free Flow at M = 2.9: 1, Calculated according to formula $St_w = 0.332 Re_w^{-1/2} Pr_w^{-2/3}$; 2, Calculated according to formula $St_w = A Re_w$; 3, Calculated according to formula $St_w = 0.029 Re_w^{-0.2} Pr_w^{-0.6}$

The influence of roughness of the surface on heat exchange has been less thoroughly studied than its influence on friction, even in an incompressible boundary layer. Here, the form of the roughness elements has not yet been successfully eliminated from analysis; due to this, the experimental dependences of various authors agree poorly with each other, even for flow in tubes. Heat exchange on the rough surface continues to depend on the Reynolds number in the area of the square law of resistance, where coefficient C_f does not depend on Re [35]. This indicates that the Reynolds analogy is disrupted for rough surfaces.

Experiments performed at supersonic velocities have shown that large roughnesses increase heat fluxes by factors of 1.5-2, and further increases in roughness height do not change heat exchange coefficient α (Figure 11). It is interesting to note that when rough and smooth sectors of the surface alternate, the value of α is determined entirely by the local conditions. Figure 11 shows values of α produced on smooth surface sectors (the size of the transducer was d ≈ 0.02 m, corresponding to 2-3 times the thickness of the boundary layer) which followed a rough surface (x = 0.15-0.5 m). We can see that the experimental values agree well with the calculated values for a smooth surface. A similar result for friction in an incompressible boundary layer on a rough surface was produced in work [36]; it is also mentioned in book [32].

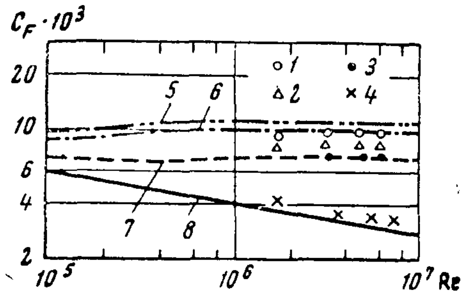

Figure 9. Influence of Roughness of Surface on Mean Value of Coefficient of Friction C_f over a plate of Length l in an Incompressible Turbulent Boundary Layer: 1, Experimental, l/h = 419, M = 0.7 [33]; 2, Experimental, l/h = 650, M = 0.7 [33]; 3, Experimental, l/h = 1680, M = 0.7 [33]; 4, Smooth plate M = 0.7 [33]; 5, Calculated, l/h = 410(?) M = 0 [32]; 6, Calculation, l/h = 650, M = 0 [32]; 7, Calculation, l/h = 1680, M = 0 [32]; 8, Smooth plate, M = 0 [45?]

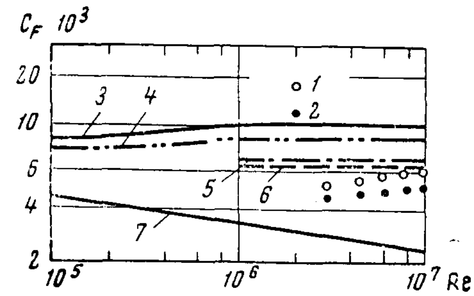

Figure 10. Influence of Surface Roughness on Mean Value of Coefficient of Friction C_F on Plate of Length l in Compressible Turbulent Boundary Layer: 1, Experimental, l/h = 450, M = 2.0 [33]; 2, Experimental, l/h = 720, M = 2.0 [33]; 3, Calculated, l/h = 450, M = 0 [32]; 4, Calculated, l/h = 720, M = 0 [32]; 5, Calculated, l/h = 450, M = 2.0 [35]; 6, Calculated, l/h = 720, M = 2.0 [34]; 7, Smooth plate M = 2.0 [45]

V. Boundary Layer on a Permeable Surface /18

The boundary layer on a porous surface with coolant flow through the surface has been investigated by many authors (for example, [37-40]). A number of empirical and semi-empirical methods for calculation have been suggested [5, 39, 41-43].

Blowing of gas through the surface leads to a considerable deformation of the velocity profiles which, with large blowing parameters B_0, take on an S-shaped form [40]. Figure 12 shows that the experimental data of various authors on blowing of air into air agree with the calculated dependences of the heat exchange coefficient on the blowing parameter with small values of this parameter ($B_0 < 1$). However, here also we see considerable divergence. As B_0 is increased, the influence of blowing indicated by most calculation methods is artificially high. Work [39] suggests an empirical dependence for the consideration of the influence of blowing of various gases on heat exchange at supersonic speeds ($0 \leqslant M_1 \leqslant 8$):

$$\frac{q_w}{q_{w0}} = 1 - 0{,}19\left(\frac{m_1}{m_v}\right)^b B_0, \tag{8}$$

where q_w, q_{w0} are the heat fluxes with and without blowing, $b = f(m_1/m_v)$, m_v and m_1 are the molecular weights of the gases. This dependence agrees well with the results of experiments for values of $q_w/q_{w0} \geqslant 0.3$, $T_w/T_e = 0.6\text{-}1.2$; $Re = 10^5\text{-}10^7$; $m_v = 2\text{-}121$.

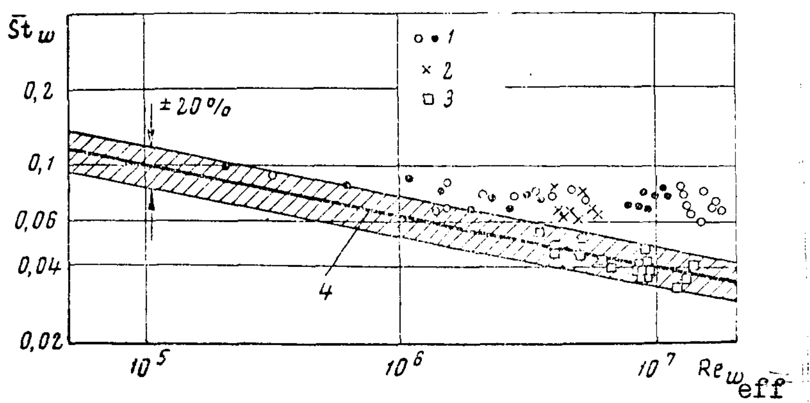

Figure 11. Influence of Large Scale Roughness of Surface on Heat Exchange Coefficient in Compressible, Turbulent Boundary Layer on Flat plate: 1, Experimental, M = 2.4, distance from beginning of boundary layer x = 10-530 mm, height of roughness h = 1 mm; 2, Experimental, M = 2.7, x = 370-530 mm, h = 1 mm; 3, Experimental, M = 2.4, x = 150-500 mm. Upstream from the point of measurement, the surface is rough (h = 1 mm). The surface of the heat exchange transducer is smooth (d = 20 mm); 4, Calculation for smooth surface according to formula

$$\overline{\mathrm{St}} = \frac{\mathrm{St}_w}{0{,}029\,\mathrm{Pr}_w^{-0{,}6}(T_w/T_e)^{0{,}39}\left(1+\frac{k-1}{2}rM_1^2\right)^{0{,}11}} = \mathrm{Re}_{w\,\mathrm{eff}}^{-0{,}2}$$

For the case of large flow rates of gas, no heat exchange calculation methods are currently available.

Everything stated above concerning the influence of blowing is correct for gradient-free flows. Where $dp/dx \neq 0$, no reliable experimental data are available on the influence of blowing: correspondingly, there are also no calculation methods. In this case, deformation of the velocity profile has an essential influence on all characteristics of the boundary layer. There is great interest in the problem of the development of the boundary layer beyond the blowing sector as the blowing intensity is changed along the length of a plate.

VI. Boundary Layer with Variable Conditions at the Surface /19

This class of problems includes the calculation of a boundary layer on a surface with variable length of roughness and blowing parameters which vary along the length of the surface, which were partially analyzed above, as well as calculation of the boundary layer with variable surface temperature T_w.

Heat exchange on the surface with a sharp change in temperature ("temperature step") in an incompressible liquid is described well by the following formula, produced under relatively simple initial assumptions [44, 45]:

$$\mathrm{St}(x,l) = 0{,}0296\,\mathrm{Re}^{-0{,}2}\left(\frac{T_w}{T_{01}}\right)^{-0{,}4}\left[1-\left(\frac{l}{x}\right)^{9/10}\right]^{-1/9} = \mathrm{St}(x,0)\left[1-\left(\frac{l}{x}\right)^{9/10}\right]^{-1/9}, \tag{9}$$

where x, l are the distances from the beginning of the boundary layer to the section being analyzed and to the point of the sudden change in temperature respectively, T_w, T_{01} are the temperatures of the wall and the flow at cross section x.

The so-called superposition method has been successfully used to calculate heat exchange on a surface with a smooth change in surface temperature; the essence of this method is that the smooth change in temperature is replaced by a stepwise change, the influence of each step being analyzed individually according to the formula used above, and the influence of all "steps" being added for the plate section in question. After the obvious transformations, the calculation formula for local heat flux on the surface with variable temperature takes on the form [45]:

$$q = \rho_1 u_1 c_p \,\mathrm{St}(x, 0) \int_0^x \left[1 - \left(\frac{\xi}{x}\right)^{9/10}\right]^{-1/9} \frac{dT}{d\xi} d\xi. \tag{10}$$

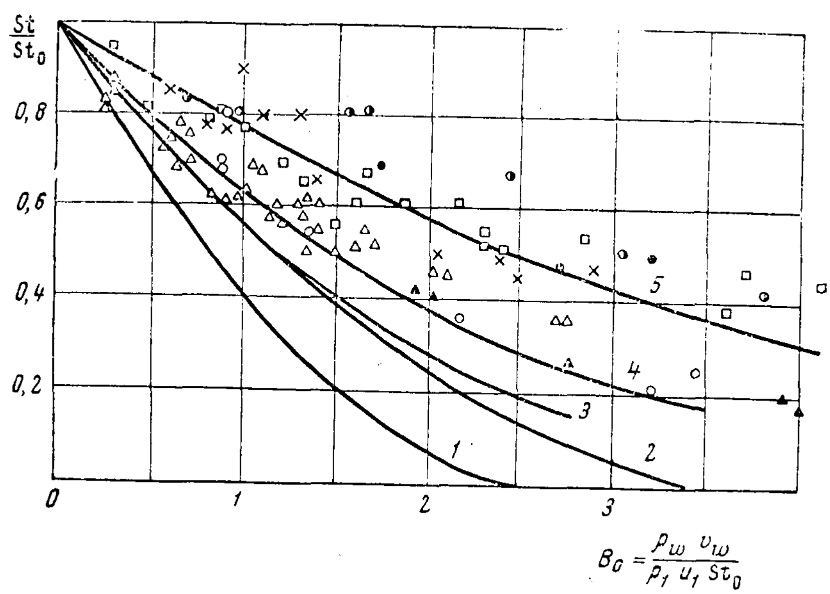

Figure 12. Influence of Blowing of Air on Heat Exchange on Plate with $T_w/T_e = 1$ and Re = const.

Most experimental points of various authors produced at $M_1 \approx 0$: 1, Calculation according to [12]; 2, Calculation according to [5] (limit law); 3, Calculation according to [41]; 4, Calculation according to theory of Rubezin [38]; 5, Calculation according to [37]

Figure 13 shows the influence of the stepped change in temperature of the surface on the coefficient of friction and heat exchange [46], averaged over a certain length d (size of the transducer) at M = 5. According to the formulas for an incompressible liquid, the ratio of the heat exchange coefficients at the transducer in the presence of a "temperature step" $\bar{\alpha}$ and at constant temperature α can be represented in the form [47] /20

$$\overline{\alpha}/\alpha = F(x/l) + zH(x/l), \tag{11}$$

where H and F depend only on the geometric dimensions, while $z = (T_{tr} - T_w)/(T_{tr} - T_{01})$. Under the conditions of the experiment whose results are shown on Figure 13, the value of $H_{incom} = 0.5$. The experimental points lie on a line described by (11), but with values of $H \approx 1.4\text{-}1.7$, i.e. in a compressible boundary layer, a sudden change in surface temperature influences the heat exchange more strongly than in an incompressible boundary layer. Attempts to extend the methods for calculating heat exchange with variable surface temperature to a compressible boundary layer have as yet been unsuccessful.

Figure 13 also shows the results of measurement of friction with a "temperature step." We see that the sudden change in temperature has practically no influence on friction.

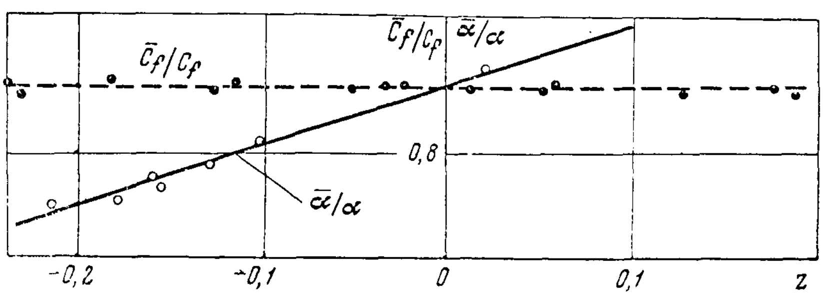

Figure 13. Influence of Sudden Change in Temperature of Surface on Values of Heat Exchange Coefficient $\overline{\alpha}$ and Friction Coefficient $\overline{C}_f$ in Compressible Layer Averaged over Length of Transducer [46]. According to [47], for the conditions of this experiment [46], $F(x/l) \approx 1$, $H(x/l) \approx 0.5$

VII. Certain Paths for Further Development of the Methods for Calculating a Turbulent Boundary Layer

Thus, the usage of semi-empirical methods for calculating (like the empirical formulas) produces reliable results only with values of defining parameters for which these dependences have been tested experimentally. Extension of the empirical or semi-empirical dependences to the limits of available experimental data can lead to erroneous results (for example, the dependence of C_f on the Mach number at $M > 9$, the dependence of heat exchange on air blown into the stream at high blowing parameters, etc.). Semi-empirical methods cannot consider any specific features in the influence of the defining parameters, which were not included in the composition of these methods. The semi-empirical methods, like the experimental dependences, are in most cases a convenient method of interpolation of the influence of the determining parameters on the primary characteristics of the flow in the boundary layer.

Therefore, the main task is not clarification of these methods in their present form, but rather improvement of the physical ideas on which they are based; the experimental determination of specific features of the influence of various parameters on the development of the boundary layer and the usage of deeper physical regularities for the flow in the turbulent boundary layer in /21
order to create new methods for its calculation in place of the commonly used resistance rule, distribution of shear stress through the thickness of the layer, etc.

From this point of view, there is interest in the work of Iyevlev, Fox and Zakkay, Bradshow et al. [9-12]. Most of these works are based on widespread usage of electronic digital computers to solve the complex differential equations in partial derivatives using experimental data on turbulent transfer coefficients.

Figure 14 shows the results [11] of calculation of C_f in an incompressible boundary layer on a smooth plate with various values of initial turbulence $\sqrt{e_0}$. Work [11] contains the numerical solution of a system of equations of movement, discontinuity and energy of turbulent pulsations

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{12}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{13}$$

$$u\frac{\partial e}{\partial x}+v\frac{\partial e}{\partial y}=\frac{\partial}{\partial y}\left(D\frac{\partial e}{\partial y}\right)+(\mu-1)\left(\frac{\partial u}{\partial y}\right)^2-\frac{CD}{L^2}e, \tag{14}$$

where the viscosity μ and diffusion coefficient D are experimental functions of the Reynolds number $\sqrt{eL}/\nu$ in the scale of turbulence L. The experimental dependence $L = f(y/\delta)$ was also used for the scale of turbulence. The solution of this system of equations is used to develop a friction rule (Figure 14). The calculated profiles agree well with experimental velocity profiles.

In essence, this method is also based on certain empirical dependences, but these dependences are more general in nature than those which are used in the semi-empirical calculation methods (resistance rule, etc.), and are generally applicable only to a narrow class of flows. Therefore, the calculation methods like those presented in [10, 11], encompass a broader class of flows and, possibly, can be used to find a solution to the problems stated above.

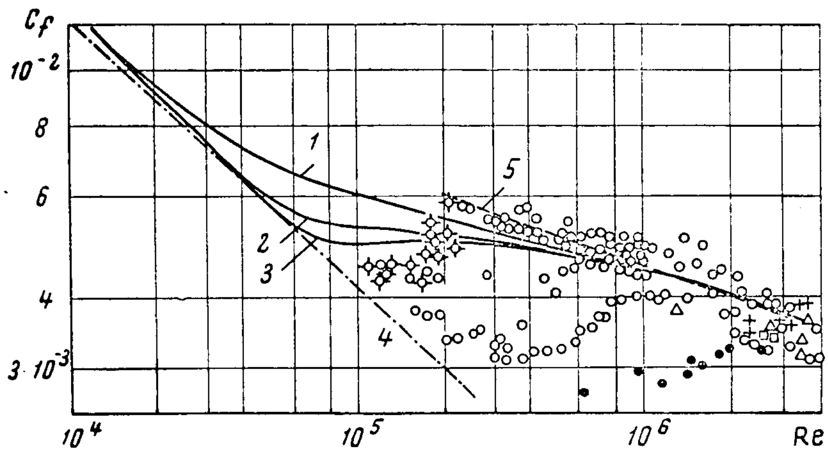

Figure 14. Calculation of Coefficient of Friction C_f with Various Initial Levels of Turbulence $\sqrt{e_0}$; 1, Calculation for $\sqrt{e_0} = 0.1$; 2, Calculation for $\sqrt{e_0} = 0.001$; 3, Calculation for $\sqrt{e_0} = 0.0001$; 4, $C_f = f(\mathrm{Re})$ for laminar boundary Layer; 5, $C_f = 0.455(\lg \mathrm{Re})^{-2.58}$ for turbulent boundary layer

REFERENCES

1. von Karman, T.: "Turbulence and Skin Friction,"J. Aeronaut. Sci., Vol. 1, No. 1, 1934.
2. van Driest, E. R.: "The Turbulent Boundary Layer for Compressible Fluids,"J. Aeronaut. Sci., Vol. 18, No. 2, 1951.
3. Wilson, R. E.: "Turbulent Boundary Layer Characteristics at Supersonic Speeds - Theory and Experiment," J. Aeronaut. Sci., Vol. 21, No. 7, 1954.
4. Kozlov, L. V., "The Relationship Between Aerodynamic Heating and Surface Friction," *Izvestiya Akademii Nauk SSSR, Mekhanika i Mashinostroyeniye*, No. 4, 1963. /23
5. Kutateladze, S. S., A. I. Leont'yev, *Teploobmen i Treniye v Turbulentnom Pogranichnam Sloye* [Heat Exchange and Friction in a Turbulent Boundary Layer], Novosibirsk, 1964.
6. Avduyevskiy, V. S., "Method of Calculating Spatial Turbulent Boundary Layer in Compressible Gas," *Izvestiya Akademii Nauk SSSR, Mekhaniki i Mashinostroyeniye*, No. 4, 1962.
7. Loytsyanskiy, L. G., *Mekhanika Zhidkosti i Gaza* [Mechanics of Liquids and Gases], Fizmatgiz. Press, 1957.
8. Kalikhman, L. Ye., "The Gas Dynamic Theory of Heat Transfer," *Prikladnaya Matematika i Mekhanika*, Vol. 10, No. 4, 1946.
9. Iyevlev, V. M., "Some Problems in the Hydrodynamic Theory of Heat Exchange in Gas Flow," *Doklady Akademii Nauk SSSR*, Vol. 86, No. 1, 1952.
10. Fox,H., V. Zakkay and R. Sinhe: "A Review of Some Problems in Turbulent Mixing," New York University Bronx., NYU-AA-66-63, 1966.
11. Glushko,G. S., "Turbulent Boundary Layer on a Flat Plate in an Incompressible Fluid," *Izvestiya Akademii Nauk SSSR, Mekhanika*, No. 4, 1965.
12. Bradshow, D. H. Ferriss and N. P. Atwell: "Calculation of Boundary Layer Development Using the Turbulent Energy Equation" J. of Fluid Mechanics, Vol. 28, 1967.
13. Chapman, D. and R. Kester: "Measurements of Turbulent Skin Friction on Cylinders in Axial Flow at Subsonic and Supersonic Velocities," J. Aeronaut. Sci., Vol. 20, No. 7, 1953.
14. Brinich, P. and N. Diaconis: "Boundary-Layer Development and Skin Friction at Mach Number 3.05," NACA TN-2742, 1952.
15. Coles, D.: "Measurement of Turbulent Friction on a Smooth Flat Plate in Supersonic Flow," J. Aeronaut. Sci. Vol. 21, No. 7, 1954.
16. Lobb, K. R., E. M. Vinkler and J. Persh: "Experimental Investigation of Turbulent Boundary Layers in Hypersonic Flow," J. Aeronaut. Sci., Vol. 22, No. 1, 1955.
17. Hill, F.: "Turbulent Boundary Layer Measurements at Mach Number From 8 to 10. The Physics of Fluids," Vol. 2, No. 6, 1959.
18. Ludvieg, H. and W. Tillmann: "Research on the Sheer Stress in Turbulent Friction Layers," ZAMM, Vol. 29, 1949.
19. Fedyayevskiy, K. K.: "Critical Review of Work on Retarded and Accelerated Boundary Layers," *Teknicheskiye Zametki TsAGI*, No. 158, 1937.
20. Sergiyenko, A. A. and V. K. Gretsov: "Transition of a Turbulent Boundary Layer to a Laminar Flow," *Doklady Akademii Nauk SSSR*, Vol. 125, No. 4, 1959.

21. Back, L. H., P. F. Massier and R. F. Cuffel: "Some Observations on Reduction of Turbulent Boundary Layer Heat Transfer in Nozzles", AIAA Journal, Vol. 4, No. 12, 1966.
22. Moratti, P. J. and W. M. Kays: "Heat Transfer to a Turbulent Boundary Layer With Varying Free-Stream Velocity and Varying Surface Temperature-Experimental Steady," International Journal Heat-Mass Transfer, Vol. 8, No. 9, 1965.
23. Zakkay, K. Toba and Ta-Sin Kuo: "Laminar, Transitional and Turbulent Heat Transfer After a Sharp Convex Corner," AIAA Journal, Vol. 2, No. 8, 1964.
24. Zisina-Mohozhin, L. M.: "Investigation of the Influence of Pressure Gradient on Development of Boundary Layer," *Zhurnal Tekhnicheskoy Fiziki*, Vol. 29, No. 4, 1959.
25. Sasman, P. K. and R. J. Cresci: "Compressible Boundary Layer With Pressure Gradient and Heat Transfer", AIAA Journal, Vol. 4, No. 1, 1966.
26. Michel, R.: "Resultats sur la couche limite turbulente aux grandes vitesses." Publication ONERA, No. 102, 1961.
27. Michel, R.: "Facteur agissant sur la transition de la couche limite aux vitesses hypersoniques" Int. Coun. of Aero. Sci., 3rd Congress, 1962.
28. Nagamatsu, H. T., R. E. Sheer and B. C. Graber: "Hypersonic Laminar Boundary Layer Transition on S-Foot Long 10° Cone, M_1 = 9,1 - 16," AIAA Paper No. 66-494, 1966.
29. Potter, J. L. and J. D. Whitfield: "Effects of Slight Nose Bluntness and Roughness on Boundary Layer Transition in Supersonic Flows," J. of Fluid Mechanics, Vol. 12, 1962.
30. van Driest, E. R. and J. Boison: "Experiments on Boundary Layer Transition at Supersonic Speeds," J. Aeronaut. Sci., Vol. 24, No. 12, 1957.
31. Nikuradse, J.: "Stromungsgesetze in rauhen Rohren," Forschg.-Arb. Ing.-Wes., No. 361, 1933.
32. Shlikhring, G.: "*Teoriya Pogranichnago Sloya*" [Theory of the Boundary Layer], Foreign Literature Press, 1956.
33. Goddard, P. E.: "Effect of Uniformly Distributed Roughness on Turbulent Skinfriction Drag at Supersonic Speeds,: J. of the Aero/Space Sciences, Vol. 26, No. 1, 1959.
34. Fenter, F. W.: "Analysis and Direct Measurement of the Skin Friction of Uniformly Rough Surface at Supersonic Speeds,: Institute of the Aeronautical Sciences, Preprint. No. 837, 1958.
35. Owen, P. R. and W. R. Thomson: "Heat Transfer Across Rough Surfaces," J. of Fluid Mechanics, Vol. 15, No. 3, 1963.
36. Jacobs, W.: "Umformung eines turbulenten Geschwindigkeits Profils, ZAMM, 1939.
37. Friedman, J: "A Theoretical and Experimental Investigation of Rocket - Motor Sweat Cooling," ARS Journal, No. 79, 1949. /24
38. Bartl, E. R. and B. M. Leadon: "Experimental Evaluation of Heat Transfer with transpiration Cooling in a Turbulent Boundary Layer," J. of the Aero/Space Sciences, Vol. 27, No. 1, 1960.
39. Mugalev, V. P.: "Experimental Investigation of Turbulent Boundary Layer on Porous Surface with Blowing of Various Gases and Variable Mach Number of Flow," *Doklady na II Vses. S'yezde po Teoret. i. Prikladn., Mekhan.* [Reports of II All-Union Congress on Theoretical and Applied Mechanics], Moscow, 1964.

40. Mugalev, V. P.: "Experimental Investigation of Subsonic Turbulent Boundary Layer on Plate with Blowing," *Investiya VUZov MVO SSSR, Seriya Aviastionnaya Teknika*, No. 3, 1959.
41. Dorrance, W. H. and F. J. Dore: "The Effect of Mass Transfer on the Compressible Turbulent Boundary - Layer Skin Friction and Heat Transfer," J. Aeronaut. Sci., Vol. 21, No. 6, 1954.
42. Kalikhman, L. Ye.: "Turbulent Boundary Layer of Incompressible Fluid on Porous Plate," *Zhurnal Teknicheskoy Fiziki*, Vol. 15, No. 11, 1945.
43. Lapin, Yu. V.: "Friction and Heat Exchange in a Compressible Turbulent Boundary Layer on a Plate with Material Injection," *Zhurnal Tekhnicheskoy Fiziki*, Vol. 30, No. 8, 1960.
44. Scesa, S.: "Experimental Investigation of Connective Heat Transfer to Air From a Flat Plate with a Stepwise Discontinuous Surface Temperature," M. S. Thesis, University of California, Berkeley, Calif., 1949.
45. Eckert, E. R. and R. M. Drake: *"Teoriya Teplo- i Massobmen"* [Theory of Heat and Mass Exchange], Gosenergoizdat. Press, 1961.
46. Westkaemper, J. C.: "Step-Temperature Effects on Direct Measurement of Drag," AIAA Journal, Vol. 1, No. 7, 1963.
47. Rubesin, M. W.: "The Effect of an Arbitrary Surface Temperature Variation Along a Flat Plate on the Convective Heat Transfer in an Incompressible Turbulent Boundary Layer," NACA TN 2345, 1951.

Translated for the National Aeronautics and Space Administration under Contract No. NASw-1695 by Techtran Corporation, P.O. Box 729, Glen Burnie, Md. 21061